Analyzing the existence of magnetoroton excitations in magnetized quantum wires

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We report on the theoretical investigation of magnetoplasmon excitations in a quantum wire characterized by a confining harmonic potential and in the presence of a perpendicular magnetic field. The problem involves two length scales: $l_0=\sqrt{\hbar/m^*\omega_0}$ and $l_c=\sqrt{\hbar/m^*\omega_c}$, which characterize the relative strengths in the interplay of confinement and the magnetic field. We embark on the charge-density excitations within a two-sub-band model in the framework of Bohm-Pines' random-phase approximation. The main focus of our study is the (inter-sub-band) magnetoroton excitation which changes the sign of its group velocity twice before merging with the respective single-particle continuum. We analyze the terms and conditions within which the magnetoroton excitation persists in the quantum wires. It is suggested that the electronic device based on such magnetoroton modes can act as an *active* laser medium.

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Scientists have long thought the way the basic notions would change in systems with dimensionality different from that of the three-dimensional (3D) space we are so accustomed to. The discovery of the quantum Hall effects[1] is known to have spurred the efforts to see the consequent changes with the reduction in the system's dimensionality from three to two, two to one, and one to zero. These are, respectively, the quasi-two-dimensional, quasi-one-dimensional (Q1D), and quasi-zero-dimensional semiconductor structures in which the charge carriers are constrained in one, two, and three dimensions or are allowed free motion in two, one, and zero dimensions.[2] The middle of this rainbow represents the so-called quantum wires or (more realistically) quasi-one-dimensional electron gas (Q1DEG) for broader range of physical understanding.

Much of the fundamental theoretical understanding of electron dynamics in one-dimensional (1D) systems have emerged from the work on the Tomonaga-Luttinger liquid model (TLLM).[3] The TLLM makes some of the drastic simplifying assumptions, which allow one to solve the interacting problem completely. One of the surprising results that is obtained from the solution of TLLM is that even the smallest interaction results in a disappearance of the Fermi surface, leading to a system which is describable as a non-Fermi liquid—in the sense that the elementary excitations are very different from those of the noninteracting system. Therefore, one would expect that the experimental properties of the semiconductor quantum wires should be quite different from any predictions based on the assumption that a one-dimensional electron gas (1DEG) is a Fermi liquid. Yet, the contrary observations have been rather firmly established.[4]

An early motivation behind the proposal of semiconductor quantum wire structures was the suggestion[5] that 1D k-space restriction would severely reduce the impurity scattering, thereby substantially enhancing the low-temperature electron mobilities. As a result, the technological promise that emerges is the route to faster transistors and optoelectronic devices fabricated out of the quantum wire structures. Research interest burgeoned in quantum wires owes not only to their potential applications, but also to the fundamental physics involved. For instance, they have offered us an excellent, unique opportunity to study the real 1D Fermi gases in a relatively controlled manner.[2]

The present Brief Report aims at investigating the charge-density excitations in a realistic quantum wire within a two-sub-band model in the framework of Bohm-Pines' random-phase approximation (RPA).[2] The main focus of our study is the (inter-sub-band) magnetoroton excitation which changes the sign of its group velocity twice before merging with the respective single-particle continuum. We analyze the terms and conditions within which the magnetoroton excitation persists in the quantum wires. A roton is an elementary excitation whose dispersion relation shows a linear increase from the origin, but exhibits first a maximum, and then a minimum in energy as the momentum increases. Excitations with momenta in the linear region are called phonons; those with momenta near the maximum are called maxons; and those with momenta close to the minimum are called rotons.

A roton mode in 3D superfluid ^{4}He was empirically derived within the two-fluid model by Landau,[6] and its reliable theory was developed and refined by Feynman.[7] In two-dimensional electron gas (2DEG), the magnetoroton minimum was obtained in the fractional quantum Hall effect regime within the framework of single-mode approximation (SMA) by Girvin *et al.*[8] In Q1DEG, the magnetoroton mode was predicted within the framework of Hartree-Fock approximation in 1992 (Ref. 9), and it was soon verified in the resonant Raman scattering experiments.[10] Here, we are concerned with the magnetoroton (MR) excitation in a 2DEG in the presence of a confining harmonic potential (oriented along the x direction) and an applied perpendicular (to the x-y plane) magnetic field B. The resultant system—a realistic Q1D quantum wire with free propagation along the y direction and magnetoelectric quantization along x—is characterized by the eigenfunction

$$\psi_n(k_y)=\frac{1}{\sqrt{L_y}}e^{ik_y y}\phi_n(x+x_c), \tag{1}$$

where $\phi_n(x+x_c)$ is the Hermite function, and the eigenenergy

$$\epsilon_{nk_y} = \left(n + \frac{1}{2}\right)\hbar\tilde{\omega} + \hbar^2 k_y^2/(2m_r), \qquad (2)$$

where L_y, n, $x_c = k_y(l_d^4/l_c^2)$, $l_c = \sqrt{\hbar/(m^*\omega_c)}$, $l_d = \sqrt{\hbar/(m^*\tilde{\omega})}$, and $m_r = m^*(\tilde{\omega}^2/\omega_0^2)$ are, respectively, the normalization length, the hybrid magnetoelectric sub-band (HMES) index, the center of the cyclotron orbit with radius l_d, the magnetic length, the effective magnetic length, and the renormalized effective mass. Here the hybrid frequency $\tilde{\omega} = \sqrt{\omega_c^2 + \omega_0^2}$. The effective magnetic length l_d refers to the typical width of the wave function and reduces to the magnetic length l_c if the confining potential is zero (i.e., if $\omega_0 = 0$). In the limit of a strong magnetic field, the renormalized mass m_r becomes infinite and the system undergoes a crossover to the two-dimensional electronic system (2DES), and hence, the Landau degeneracy is recovered.

For the illustrative numerical examples, we focus on the *narrow* channels of the $In_{1-x}Ga_xAs$ system. We compute the magnetoplasmon excitations in a Q1DEG within a two-sub-band model in the presence of a perpendicular magnetic field B at T=0 K. We do so by examining the influence of several parameters involved in the analytical results. These are, for instance, the 1D charge density n_{1d}, characteristic frequency of the harmonic potential ω_o, and the magnetic field B. The material parameters used are: effective mass $m^* = 0.042 m_0$, the background dielectric constant $\epsilon_b = 13.9$, 1D charge density $n_{1D} = 1.0 \times 10^6$ cm^{-1}, confinement energy $\hbar\omega_0 = 2.0$ meV, and the effective confinement width of the harmonic potential well, estimated from the extent of the Hermite function, $w_{\text{eff}} = 40.19$ nm. Notice that the Fermi energy ϵ_F varies in the case where the charge density (n_{1D}), the magnetic field (B), or the confining potential ($\hbar\omega_0$) is varied.

The magnetoplasmon spectrum within a two-sub-band model using the full RPA was illustrated in Fig. 9 of Ref. 11. We call attention to the most curious part of that excitation spectrum—the existence of the inter-sub-band collective (magnetoplasmon) excitation (CME) [henceforth referred to as the MR], which changes the sign of its group velocity twice before merging with the respective single-particle excitation (SPE). The interesting thing about its very occurrence in Q1DEG (Ref. 11) leads us to infer that you do not have to overplay with the theory, as was done in Refs. 12 and 13, which both missed to observe this MR mode. The said magnetoroton excitation dispersion in the energy–wave-vector space is illustrated in Fig. 1. Each MR mode corresponds to a given magnetic field and for the fixed values of the charge density (n_{1D}) and the confining potential ($\hbar\omega_0$). The important feature noticeable from Fig. 1 is that as the magnetic field (B) is increased the maxon maximum shifts to the higher energy, whereas the roton minimum first observes an increase and then (after a certain value of B, here $B = 1.5$ T) a decrease in energy.

Figure 2 shows the MR dispersion for various values of the charge density and for the given values of the magnetic field and the confinement energy. Unlike the variation in B (Fig. 1), Fig. 2 makes it evident that there is a systematic trend in the behavior characteristics of the MR as the charge density varies. To be specific, both the maxon maximum and roton minimum gradually shift to the higher energy (and longer wavelength) with increasing charge density. It is also noteworthy how the roton minimum becomes deeper (from shallower) with increasing n_{1D}.

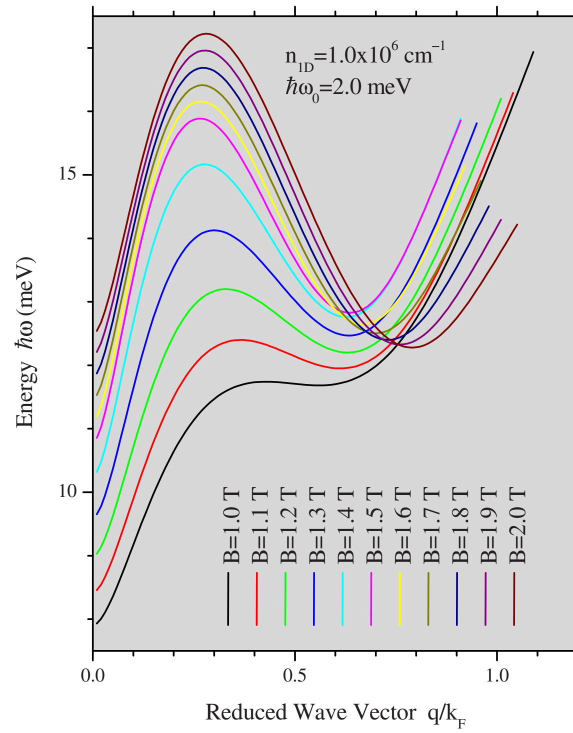

FIG. 1. (Color online) MR dispersion plotted as energy $\hbar\omega$ vs reduced wave vector q/k_F for various values of the magnetic field (B) for the given values of n_{1d} and $\hbar\omega_0$.

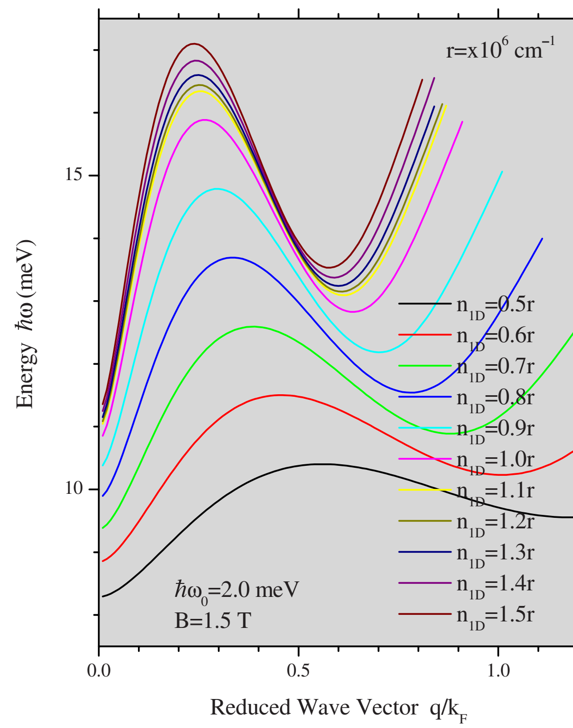

FIG. 2. (Color online) MR dispersion plotted as energy $\hbar\omega$ vs reduced wave vector q/k_F for various values of the charge density (n_{1D}) for the given values of B and $\hbar\omega_0$.

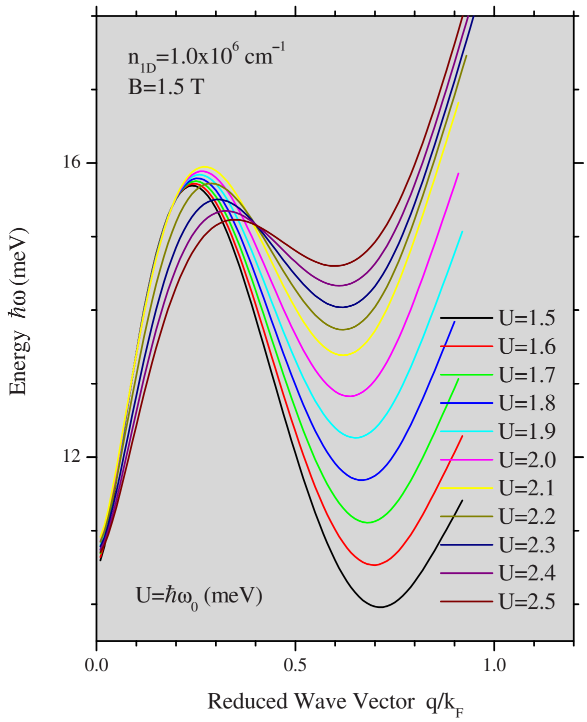

FIG. 3. (Color online) MR dispersion plotted as energy $\hbar\omega$ vs reduced wave vector q/k_F for various values of the confinement energy $(\hbar\omega_0)$ for the given values of n_{1d} and B.

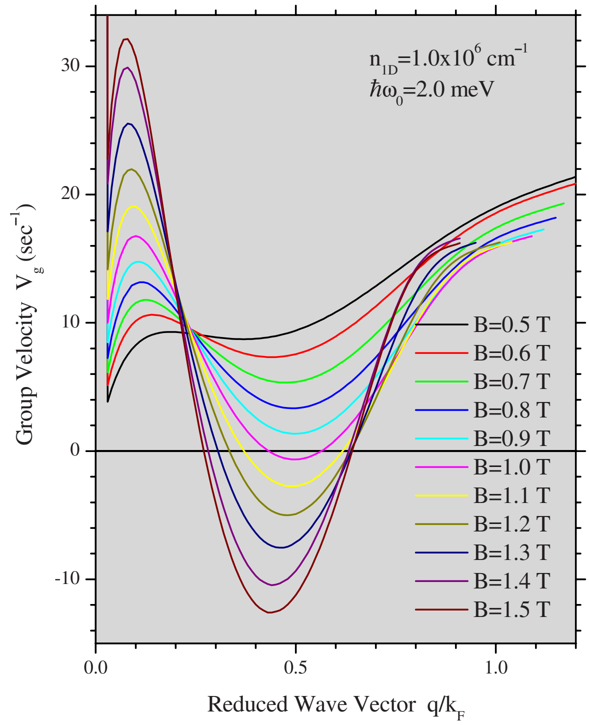

FIG. 4. (Color online) The group velocities of the MR excitations plotted in Fig. 1 as a function of the reduced wave vector q/k_F for several values of B and for the given values of n_{1d} and $\hbar\omega_0$.

Figure 3 depicts the MR dispersion for various values of confinement energy and for the given values of the charge density and the magnetic field. It is interesting to notice here that the roton minimum observes a systematic shift to the higher energy (and longer wavelength) with increasing confinement potential. On the other hand, the maxon maximum first observes an increase and then (after a certain value of $\hbar\omega_0$, here $\hbar\omega_0=2.1$ meV) a decrease in energy with increasing confinement potential. However, the maxon maximum always tends to attain a shorter wavelength with increasing $\hbar\omega_0$. A distinctive feature of Fig. 3 (as compared to Figs. 1 and 2) is that the MR mode here starts (at the origin) within a relatively narrower energy range even though the confinement potential varies. There is a common feature observed (not shown here) in all Figs. 1–3: every MR mode in the short wavelength limit merges with the upper branch of the respective (inter-sub-band) SPE.

Figure 4 illustrates the group velocities of the MR excitations (plotted in Fig. 1) as a function of the reduced wave vector q/k_F. Notice that the dimension of the group velocity is sec^{-1} because we define $V_g=\partial\omega/\partial Q$, with $Q=q/k_F$ as the reduced wave vector. One can easily notice that the inter-sub-band CME attains its magnetoroton shape only at $B \geq 1.0$ T for which values the V_g curves cross the zero twice: the first for the maxon and the second for the roton. As such, there is a minimum (threshold) value of B (i.e., $B_{\min}$) below which the MR does not exist.

Figure 5 shows the density of excitation states (DOES) of the MR mode versus the energy for several values of the magnetic field and for the given values of n_{1d} and $\hbar\omega_0$. An interesting feature that this figure dictates is that both maxon and roton are the higher density of excitation states.

In other words, the group velocity of the inter-sub-band CME becomes negative between the maxon maximum and roton minimum. An interesting feature of this aspect is that it leads to tachyon-like (superluminal) behavior without one

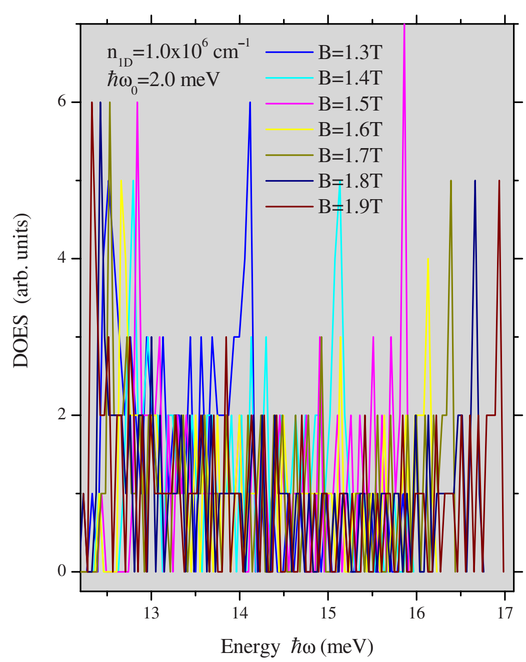

FIG. 5. (Color online) The density of excitation states (DOES) of the MR excitations plotted in Fig. 1 as a function of energy $\hbar\omega$ for several values of B and for the given values of n_{1d} and $\hbar\omega_0$.

having to introduce negative energies. The interest in negative group velocity is based on anomalous dispersion in a gain medium, where the sign of the phase velocity is the same for incident and transmitted waves and energy flows inside the gain medium in the opposite direction to the incident energy flow in vacuum. The insight is that demonstration of negative group velocity is possible in media with inverted populations so that gain instead of absorption occurs at the frequencies of interest. A medium with an inverted population has the remarkable ability of amplifying a small optical signal of definite wavelength; i.e., it can serve as an *active* laser medium. The situation is analogous to the superlattices where the crystal can exhibit a negative resistance: it can refrain from consuming energy like a resistor and instead feed energy into an oscillating circuit.

In summary, we have investigated extensively the magnetoroton excitations in the quantum wires within a two-subband model in the framework of the full RPA. The existence of the MR mode in quantum wires is solely attributed to the applied perpendicular magnetic field. We have studied dispersion characteristics of the MR as a function of several important experimental parameters such as the magnetic field, the charge density, and the confinement potential. It is observed that there is a minimum (threshold) value of the magnetic field (B_{min}) below which this MR does not exist. The roton minimum is the mode of higher density of excitation states. It is worth mentioning that roton features are among the most significant manifestations of the many-particle interactions. They arise from the interplay between direct and exchange terms of the electron gas, and the depth of the minimum is determined by the strength of the exchange vertex corrections. As such, incorporating the many-body effects adequately should give a better insight into the propagation of the magnetoroton mode. We suggest that the electronic device based on such magnetoroton modes can act as an *active* laser medium.

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